MATH 622
Mathematical Modeling I
Group Project I

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Introduction

The scenario under consideration applies to the school of Mathematical Sciences at RIT. Currently, class sizes are capped at 35 students for each section, but as the student body at RIT continues to grow, this cap may have to increase.

A greater student body implores greater demands from the school and its resources. In facing the decision of whether or not to increase class sizes, RIT must address the impact made by doing so.

First and foremost, the number of students attending classes will be the most noticeable change. Given an increase in student population, budgetary restrictions are another factor to consider, since the cost of sufficiently providing education for students is important. This cost may entail hiring more faculty and staff, along with T.A.'s, and providing more funding and scholarship assistance.

Should class sizes be enlarged, the result must maintain, or possibly even improve, student success at RIT. A decrease in passing rates would signal to the school that making this change was not a decision made in the best interest of their students.

This is a lot to consider, hence we have made it a prerogative to compare class size, rate of student success, and budget (which we take to be professor salaries) to make for a simple setup that can be expanded upon.

Presentation of the model

Given that enrollment at RIT is increasing, we need to verify if an increase in class size, CS, is the right course of action. We want to assume that the majority of the undergraduate student population is enrolled in at least one course in the School of Mathematical Sciences. Hence, total enrollment (TE) in this department is a parameter.

Please note that graduate enrollment is not included in our analysis due to differences in enrollment numbers and class sizes when compared to undergraduate data. Provided data for grad students in the School of Mathematical Sciences, our model would also be able to identify an optimum class size, tailored for success.

Passing rate, PR, is another variable we wish to quantify since it is used as a measurement of student success. It is our gracious assumption that rates of student's success are closely correlated with quality of education being provided. This implies that we have competent professors and adequate spaces to promote learning in.

While budgetary expenditure is a single parameter, there are several assumptions being made that determine this value. Professors in the School of Mathematical sciences are assumed to earn an equal amount per course taught. The additional amount of time a professor spends doing research and engaging in committee responsibilities should be included in their yearly earnings. Teaching assistant pay is not included in this category due to a negligible salary. The spaces reserved for teaching in are furnished with classroom necessities, possess sufficient technological capabilities, and are well maintained. These are the encompassing factors that summarize the total budget per course, BPC.

Cost, C, is related to budget per course (BPC), since it is the actual value that realistically demonstrates what the school will have to spend that semester on math sections. This calculation is dependent on total enrollment (TE) per class size (CS), multiplied by the budget per course (BPC).

We assume that passing rate and class size are inversely proportional, with dimensionless proportionality constant k.

$$\begin{cases} C = \frac{TE}{CS} \cdot BPC \\ PR = \frac{k}{CS} \end{cases}$$

Variable/Parameter	Dimension
Class Size CS	Students
Total Enrollment TE	Students
Passing Rate PR	Students^{-1}
Budget Per Course BPC	Money
Total Cost C	Money
Proportional Constant k	Dimensionless

Utilizing the computer program MATLAB, these variables, parameters, and equations are programmed into a script that has the capability to calculate optimal class size in two ways:

1. By creating a function where a comparison of budget and total enrollment determines passing rates
2. Using target passing rates, along with a specified budget, to determine what total enrollment numbers should be

This mathematical model is a rigorous approach to determining what the effects of increased class sizes will be, and how to go about implementing them in the most beneficial manner possible. Given a limited budget, our goal is to maximize student success. The model we have programmed is aimed at aiding the School of Mathematical Sciences in their decision by providing both thoughtful and accurate results.

Discussion of solutions

After reviewing several articles related to the matter, there were many that suggested decreasing class size for the betterment of student performance. In particular, one article [2] stated that reducing the number of students in a class from 22 to 15 increased student scores by 5%. We chose to use this range to determine our constant of proportionality, k, which resulted in a value of $k = 10.37$.

Using Glassdoor, a helpful website that gives salary estimations for many occupations, we were able to determine that each professor at RIT earns an average of roughly $\$87,000$ a year[1]. If the number of courses taught per professor is 3, then each course is worth $\$15,000$ of that salary, giving us the figure for budget per course (BPC) and simultaneously helping us to determine the total cost (C). This also allows for our assumption that the rest of this yearly income alloted is the result of research and committee obligations.

Current semester enrollment in courses in the School of Mathematical Sciences, found on Tiger Center, is 4,418 undergraduate students. Thus, this is the number used as an initial total enrollment (TE). Furthermore, about 130 was the number of sections of math that were run this semester, making 130 a reasonable number to use for estimation. Any number of sections that exceed 200 is considered to be implausible.

Using the aforementioned data values, our MATLAB code was able to generate a number of class size scenarios. The results can be seen in the following graphs. Holding the class size constant, we can see that cost has a linear relationship with the passing rate, with slope $\frac{TE \cdot BPC}{k}$. This is shown in figure (1) on page 4, for various student populations.

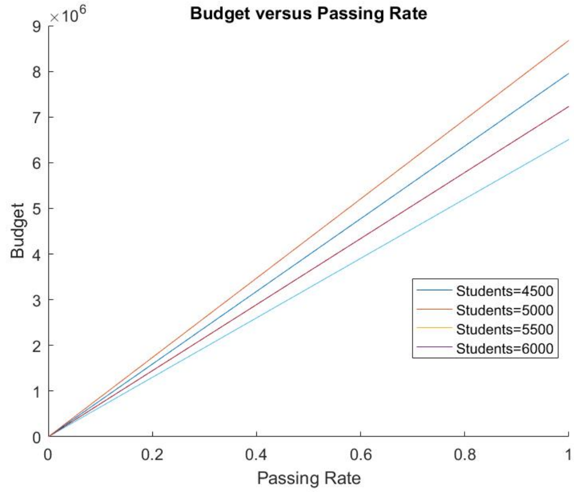

Figure 1: Plot of Budget vs. Passing rate for various student populations.

Holding the class size constant again, If we wish to maintain a given passing rate as the math student population increases, it will cost more and more money (anywhere between 1000 and 1200 dollars per additional student). This is shown in figure(2) on page 5.

To examine the effect that adding one student has on the passing rate, we took the derivative of passing rate with respect to class size: $\frac{dPR}{dCS} = \frac{-10.37}{CS^2}$. Since CS is still in the denominator, as class size increases, the effect of adding another student to the class decreases. When there are 35 people in the class, there would be a change of $\frac{-10.37}{35^2} = .008$ or decrease of 0.8%. Therefore an upper estimate for the effect of increasing the class size by 3 students would be 2.5%. This is equivalent to at most 1 fewer person passing the class since $\frac{1}{38} = 2.6\%$. If there is a dramatic increase in student demand for math classes (to 5000 students), then the class sizes may have to increase by more than 3 students, or the budget would have to increase to allow more sections to be run. The following table gives some values for various budgets, passing rates, class size, and the number of sections there would need to be.

Cost	Passing Rate	Class Size	Number of Sections
1800000	0.504	42	120
2000000	0.560	38	133
3000000	0.840	25	200
2678571.43	0.750	28	179
2857142.86	0.800	26	190
3035714.29	0.850	25	202

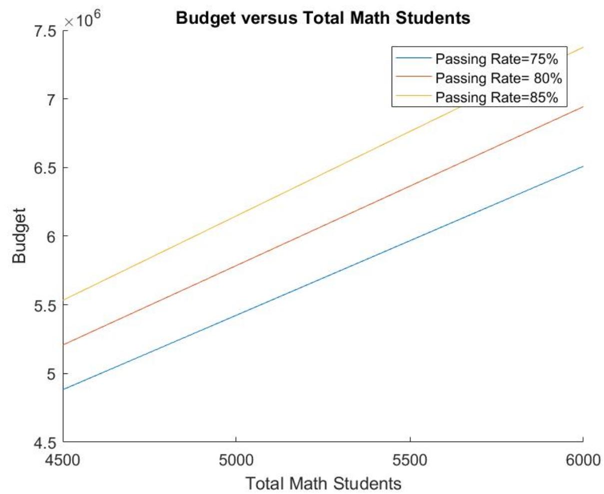

Figure 2: Plot of Budget vs. Total Math Students for various desired passing rates.

If the budget stays the same, the class size needs to increase. However, if more money is available, the students would have a better educational outcome by keeping the class size smaller.

Conclusions

After careful consideration of our calculations, and the derivative of our passing rate function, our assessment is that the addition of 1-3 students on top of a class size cap of 35 has a negligible effect on passing rate.

However, keeping this passing rate at its optimum is our goal, and too many additional students, no matter how negligible each is, add up. Thus, our suggestion is a modest one. Keep the class size increase to a minimum by only allowing a few more entrants. Given the total number of course sections RIT already offers, without altering this number, RIT would be able to accommodate 200-300 more students per section without causing classroom crowding.

References

[1] Glassdoor, 2017.

[2] Chingos, Matthew M. Whitehurst, Grover J. "Russ", *Class size: What research says and what it means for state policy* (2001).